

Inquiry and Engagement in an interactive classroom

Annalisa Crannell
Franklin & Marshall College

2018

Warning!

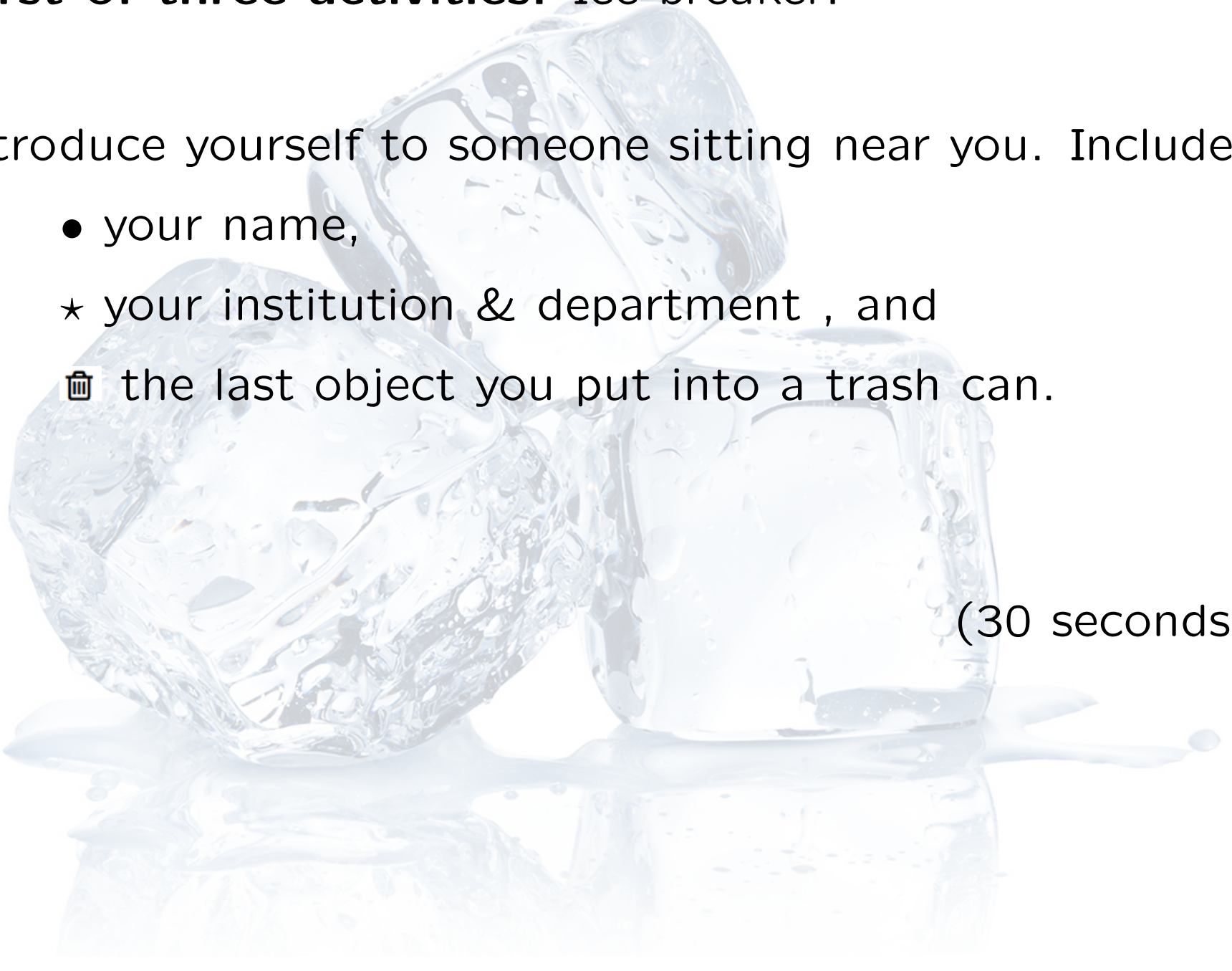
In this talk, I have included three short activities for you and people around you.

First of three activities: Ice breaker.

Introduce yourself to someone sitting near you. Include

- your name,
- ★ your institution & department , and
- 🗑 the last object you put into a trash can.

(30 seconds)





(Time's up.)

Thank you!!!



In 2017, my family (with 3–6 people at home)
put 10 trashcans at the curb.





Second of three activities:

★ Turn back to that someone sitting near you.

🗑️ Suggest two–three ways
you might have avoided that last piece of trash.
(That is, avoided adding to a landfill).

(1 minute)



(Time's up.)

Thank you again!!!

Your responses probably fell into one of these six categories:

1. R__f__s__ the trash in the first place
2. R__pl__c__ with a trash-free alternative
3. R__ __s__ durable objects vs. disposable objects
4. R__d__c__ the amount of things you use
5. R__c__cl__ instead of tossing in the landfill bin
6. R__t organic items

credit: The Zero Waste Home

1. R__f__s__: Avoid the trash in the first place



2. R__pl__c__: Change to a trash-free alternative



versus



3. Reusable durable objects, such as water bottles, bags, spoons, cloth napkins, 'xeryp' containers, etc.



4. R__d__c__ the amount of things you use



5. R__c__cl__ instead of tossing in the landfill bin



6. R__t organic items



6. R__t organic items



As my daughter once told her friends,

“My mom has a PhD *and* a compost pile!”

credit: The Zero Waste Home



Bea Johnson's 2015 trash

So . . .

. . . what does “talking trash”
tell us about teaching math?

Talking to strangers is uncomfortable for a lot of people.

Talking to friends makes it easy to get off-track.

The room set-up is bad for effective conversation & follow-up.

We come to a talk (or a class) to hear from an “expert”, not from the random person sitting next to us.

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Talking to friends makes it easy to get off-track.

Talking to strangers is uncomfortable for a lot of people.

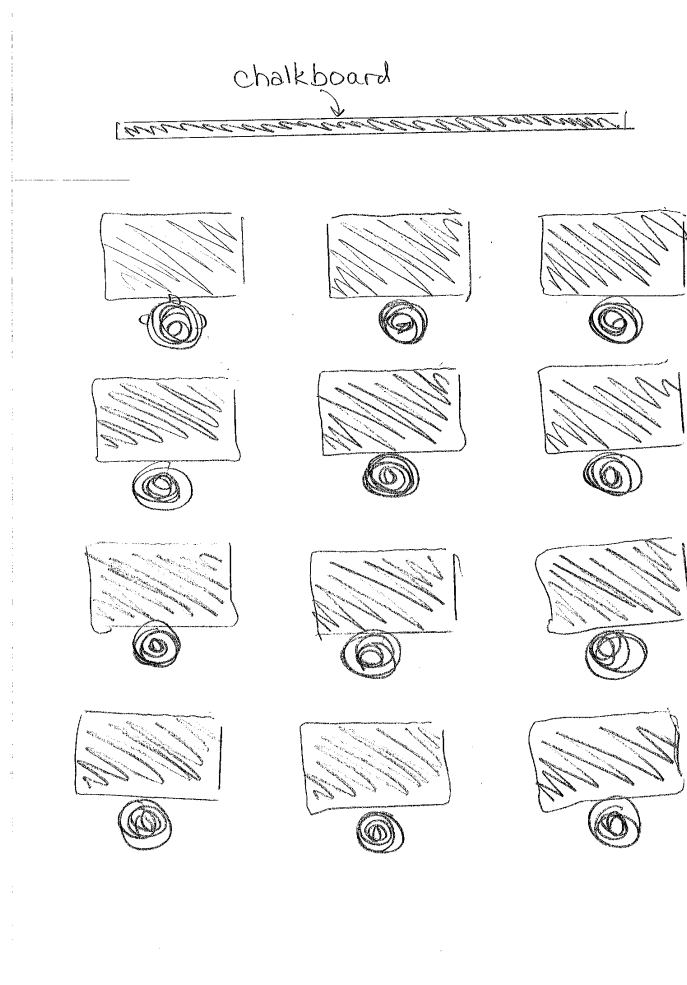
Talking to friends makes it easy to get off-track.

So make the rules specific and explicit:

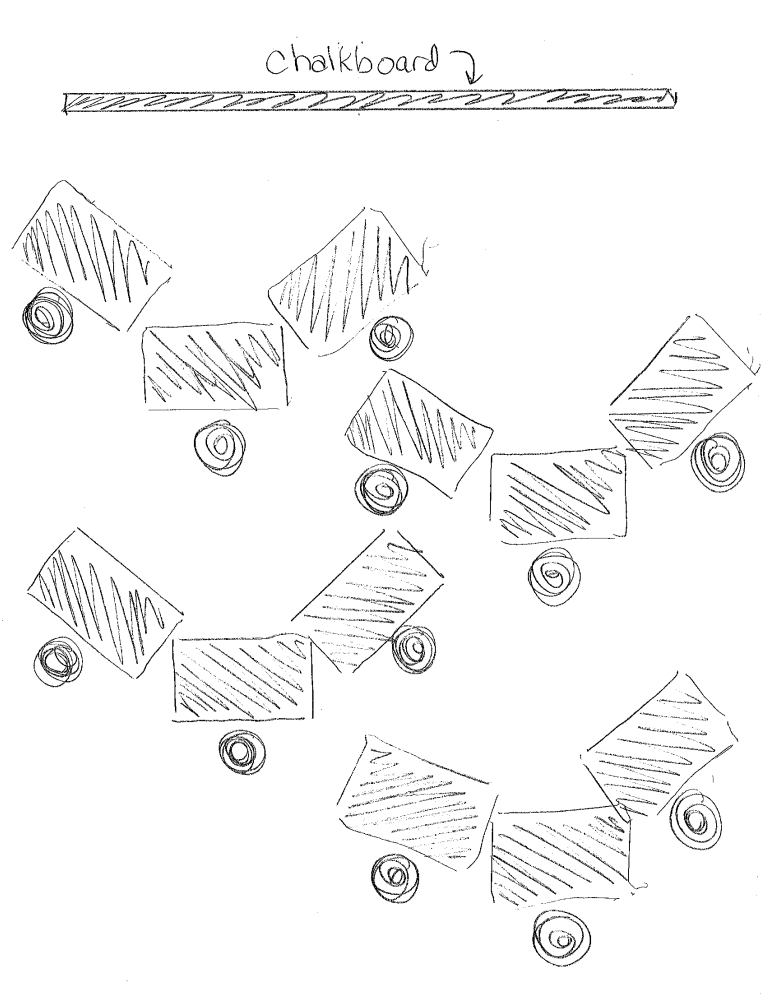
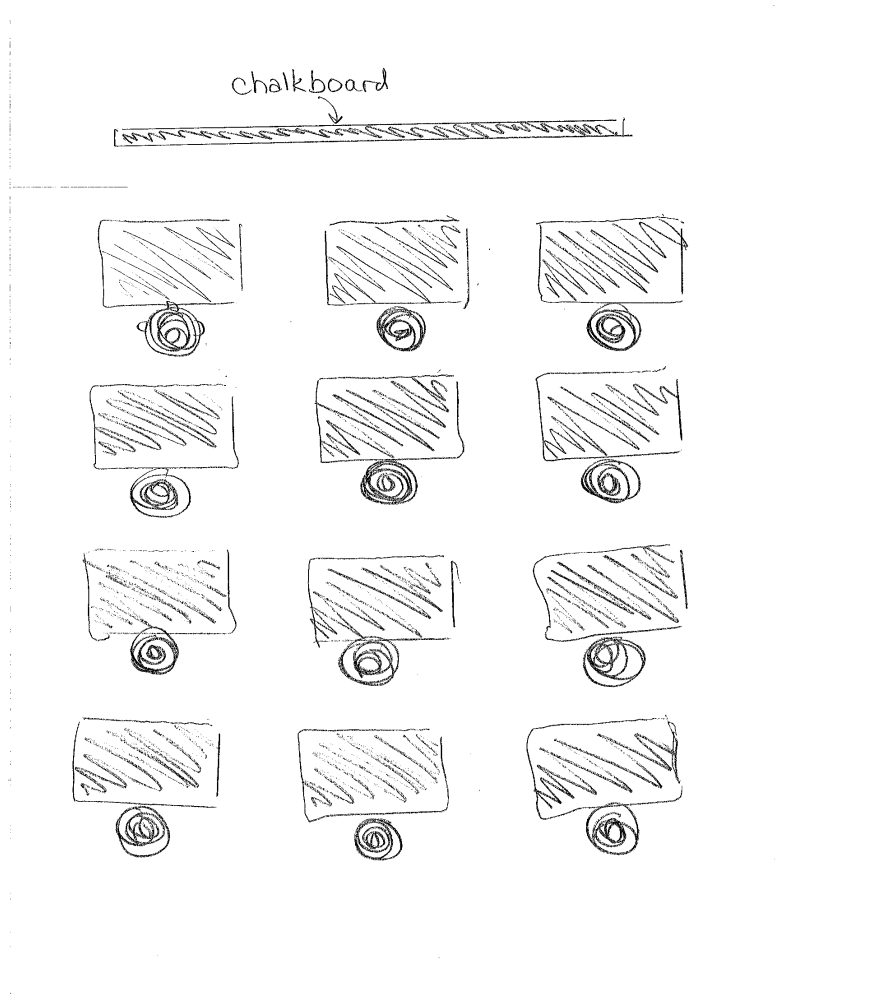
timing, roles, and scope of discussion.

**The room set-up is bad for effective conversation
& follow-up.**

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Answer 1, for the instructor:

So carefully craft questions that people can solve themselves, *with some difficulty*, together.

**We come to a talk (or a class)
to hear from an “expert”,
not from the random person sitting next to us.**

Answer 2, for the student:

But what we want, and what help us learn best,
are two different things.

What helps us learn best? *Interactive Engagement*

What helps us learn best? Interactive engagement has three components for students:

1. struggle with the problem themselves;
2. talk to other students about their approaches;

and then

3. get immediate feedback.

see Epstein, *AMS Notices* (2013).

Interactive Engagement includes
good, reasonable struggle.

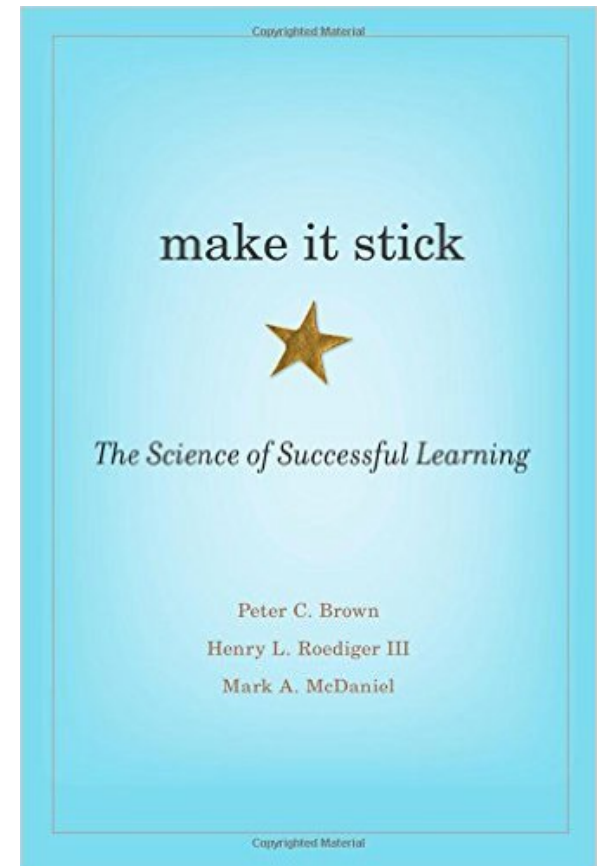
We prefer the solutions we come up with ourselves, not the ones other people give us. So introduce “desirable difficulties” :

1. *R__f__s__ the trash in the first place.*

vs.

1. *Refuse the trash in the first place.*

It is better to guess a wrong answer (and get feedback) than to make no attempt. When learners commit errors and are given feedback, the errors are not learned.



Brown, Roediger, and McDaniel (2014)

What does it mean to say *Interactive Engagement* “works better” ?

Next page:

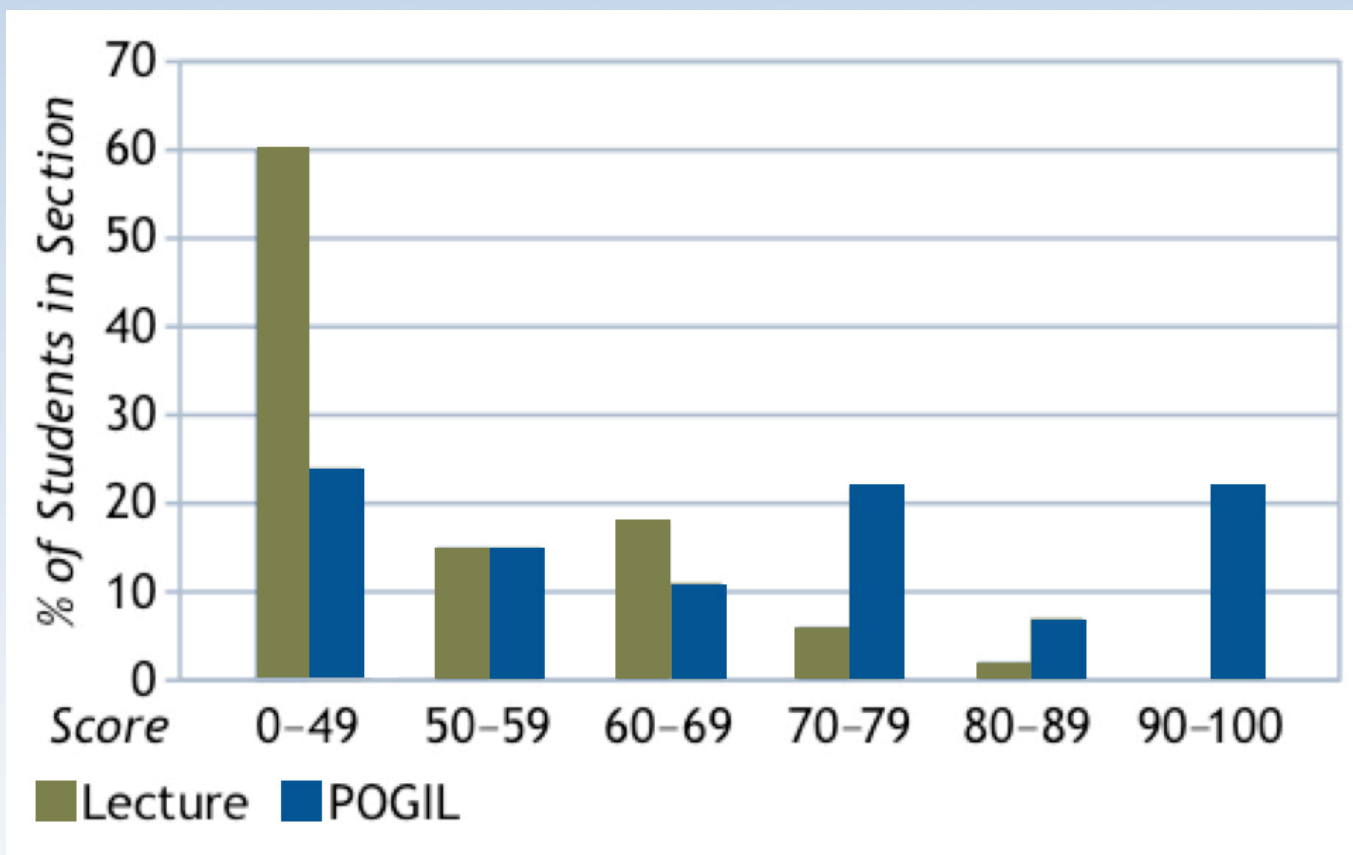
Organic Chem II student grades
on a “review” test covering Organic Chem I material.

Gray/green bars: lecture,

Blue bars: Inquiry/discovery method

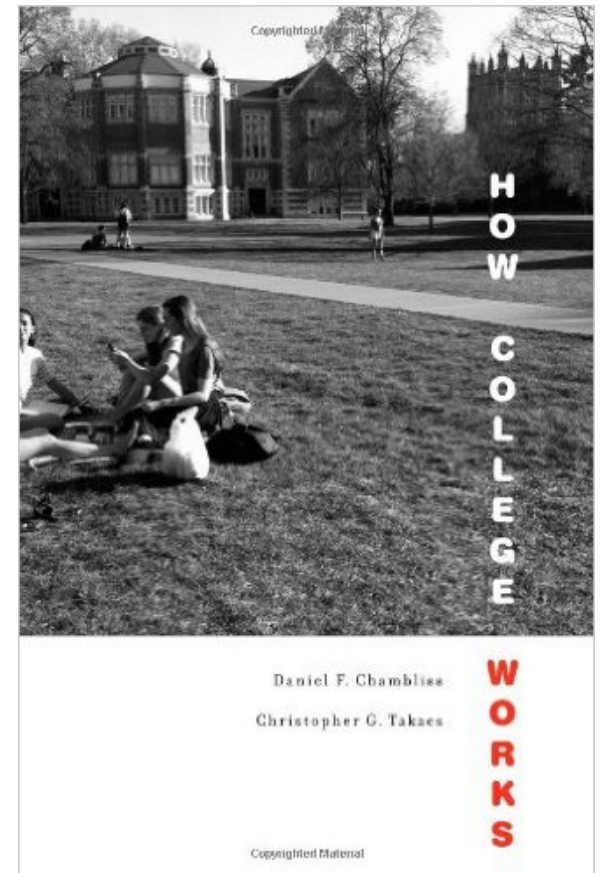
Organic Chemistry 2 Pre-Quiz at a Large Public University

Organic 2 Pre-quiz Results (Lecture vs. POGIL Organic 1)



Ruder, S.M., & Hunnicutt, S.S. (2008). POGIL in Chemistry Courses at a Large Urban University: A Case Study. In R.S. Moog, & J.N. Spencer (Eds.), *Process-Oriented Guided Inquiry Learning: ACS Symposium Series 994* (pp. 133–147). Washington, D.C.: American Chemical Society.

“College works when it provides a thick environment of constant feedback, driven by the establishment and maintenance of social relationships.”



Chambliss and Takacs, 2014.

My first (accidental) IBL class:

I was jealous of the Geology Department, so I took my students to a coffee house.





My first worksheets were masterpieces in pencil.

SQUARE ONE COFFEE HOUSE APRIL 2, 2002
 INVESTIGATING COSETS & NORMAL SUBGROUPS

EXAMPLE 1

PULL OUT YOUR HANDY-DANDY CAYLEY TABLE FOR D_3 .
 WE WILL INVESTIGATE LEFT & RIGHT COSETS OF 2 SUBGROUPS:

$$\mathcal{R} = \{0, 1, -1\} \quad \text{and} \quad \mathcal{J} = \{0, \vee\}$$

	a	a \mathcal{R}	\mathcal{R} a	a \mathcal{J}	\mathcal{J} a
0					
1					
-1					
\vee					
ρ					
\mathcal{N}					

FILL IN THIS TABLE

\mathcal{R} is a normal subgroup; \mathcal{J} is not. What do you think the definition of a normal subgroup is?

EXAMPLE 2

U_{18} , like D_3 , has subgroups of order 3 and 2.

$$U_{18} = \{ \quad \quad \quad \}$$

$$H = \{ \quad \quad \quad \} \text{ is the subgroup of order 3}$$

$$J = \{ \quad \quad \quad \} \text{ is a subgroup of order 2.}$$

List all cosets of H in U_{18} :

List all cosets of J in U_{18} :

Is H a normal subgroup of U_{18} ?

Is J a normal subgroup of U_{18} ?

PAGE 2

④ \mathcal{J} is isomorphic to a group we already know and love. What is that group?

⑤ Define a function ϕ which serves as an isomorphism from \mathcal{J} to that other group.

$$\phi\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$$

$$\phi(1-x) = \underline{\hspace{2cm}}$$

⑥ Explain why ϕ is 1-to-1.

⑦ Explain why ϕ is onto.

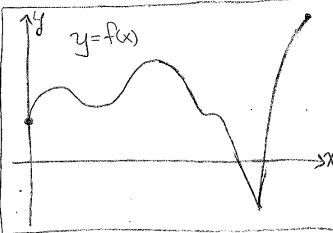
⑧ Explain why ϕ preserves group operation.

ANNOUNCEMENTS

Come to Job Candidate talks: Friday, Monday, Wednesday!
 There is no homework due Tuesday March 19.
 There's a Math Club Meeting Wednesday the 20th at 7:30, Stager 212.

I started using occasional worksheets in calculus classes, too.

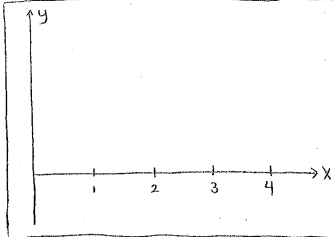
DERIVATIVES are the BEST!!
Understanding minimum, maximum, & optimization



$y=f(x)$

① The graph of $y=f(x)$ has 3 places that a local maxima, that is, places higher than other nearby points on the graph. Mark each one with a $*$.

② The graph also has 3 local minima. Mark each of those with an X .



③ Add circles ($*$ → \odot) and X → \otimes) to mark which of these are the global maximum & global minimum (the highest/lowest points of all.)

④ Now draw a function yourself that has

- x - local minima at $x=0$ & $x=2$
- $*$ - local maximum at $x=1$
- \otimes - global/local minimum at $x=4$
- \odot - global/local maximum at $x=3$. (Mark these points).

⑤ Based on these graphs you can see that max's & min's can happen at three kinds of points:

- 1) Endpoints.
- 2) Places where the derivative does not _____.
- 3) Places where the derivative equals _____.

⑥ True/False: If $f'(a)=0$, then 'a' is a max or min for f . Mark a point on the first graph w/ "T/F" that explains your answer.

book cite: "RealLife Maximum/Minimum"

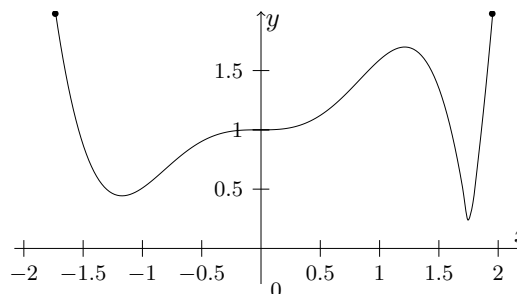
And then Max came along . . .



Now I have lots of \LaTeX -ed worksheets . . .

Critical Points and Optimization

4.2



1. The graph above has three local maxima and two local minima. Circle these five points.
2. Then fill in the blanks in the following sentence: If f is a function whose domain is the interval $[a, b]$, then local minima and local maxima happen either
 - (a) at the _____ of the interval, or
 - (b) at places where the derivative is equal to _____, or
 - (c) at places where the derivative does not _____.
3. [True or False?] If $f'(c) = 0$ for some value $x = c$, then f has a local maximum or minimum at $x = c$.
4. Suppose the volume of a box is given by $V(h) = 180h - 39h^2 + 2h^3$, where the height h of the box ranges from 0 to $\frac{15}{2}$.

... that I've posted on my web pages, for the curious

The screenshot shows a website for Annalisa Crannell, a Professor of Mathematics at Franklin & Marshall College. The page is titled "Course Materials" and features a navigation menu with options like Home, People Search Directory, Annalisa Crannell, and Course Materials. The main content area is divided into several sections:

- Annalisa Crannell**
Professor of Mathematics
717-291-4222
annalisa.crannell@fandm.edu
Office: (sabbatical)
- Talks and Presentations**
- Videos**
- Writing Projects in Math Classes**
- Course Materials**
- Contact and CV**

The "Course Materials" section includes:

- Mathematics and Art**
 - Class modules:** Projective Geometry applied to Perspective Art (an IBL intro to proofs course for math majors)
 - Course-Follow-up Blog** for Perspective Geometry
 - the book: **Viewpoints: Mathematical Perspective and Fractal Geometry in Art**
 - Instructor's Materials for a **First Year Seminar** on the math of art
- Calculus Resources**
 - Writing Projects for Calculus**
 - Guide to Writing in Math Classes**
 - IBL Worksheets for Calculus I**
 - IBL Worksheets for Calculus II**
- Worksheets for upper-level courses**
 - Algebra** (IBL worksheets that coordinate with Gallian's *Algebra*)
 - Analysis** (IBL worksheets that coordinate with Abbott's *Understanding*)

A central graphic features a thought bubble with an apple and text: "Suppose an apple falls from an extraordinarily tall tree (400 feet), and lands on the ground beside Isaac Newton, contemplating calculus. Acceleration due to gravity is constant: 9.8m/s^2 or 32ft/s^2 (downward, of course). Even when an apple is on the tree, gravity pulls downward; that's why apples dangle from the tree instead of floating like balloons around the tree."

At the bottom right, there is a book cover for "VIEWPOINTS: Mathematical Perspective and Fractal Geometry in Art" featuring a winter landscape with trees and a sunset.

Even more worksheets for IBL math classes:

Journal of Inquiry-Based Learning in Mathematics



Authors	Contents	Students	Unrefereed Notes																									
<p>Senior Editors</p> <ul style="list-style-type: none"> • Managing Editor: W. Ted Mahavier Department of Mathematics Lamar University Email: ted.mahavier@lamar.edu • Submissions Editor: David M. Clark Department of Mathematics SUNY at New Paltz Email: clarkd@newpaltz.edu • Technical Editor: Paul J. Kapitza Department of Mathematics Georgia Highlands College Email: pkapitza@highlands.edu <p>Editorial Board</p> <ul style="list-style-type: none"> • B. Dale Daniel, Lamar University • Jacqueline Jensen-Vallin, Lamar University • Judy Kennedy, Lamar University • E. Lee May, Salisbury University • Nathaniel Miller, University of Northern Colorado • G. Edgar Parker, James Madison University (Emeritus) • Carol Schumacher, Kenyon College 	<p>Journal Contents</p> <p>These refereed course notes fulfill the guidelines listed under Information for Authors.</p> <p>This table is sortable. Click on a column heading to sort by that column. Click on a title to access these notes.</p> <table border="1"> <thead> <tr> <th>Title</th> <th>Author</th> <th>Subject</th> <th>Pages</th> <th>Issue</th> </tr> </thead> <tbody> <tr> <td>Point-Set Topology</td> <td>Orloff Clark , Lisa</td> <td>topology</td> <td>28</td> <td>No. 41 (Apr. 2016)</td> </tr> <tr> <td>Topology Notes</td> <td>Person, Laura J.</td> <td>topology</td> <td>57</td> <td>No. 40 (Jan. 2016)</td> </tr> <tr> <td>Calculus I: Notes and Problems</td> <td>Stallmann, Cornelius</td> <td>calculus</td> <td>76</td> <td>No. 39 (Sept. 2015)</td> </tr> <tr> <td>Discovering Infinity</td> <td>Olinick, Michael</td> <td>set theory</td> <td>71</td> <td>No.</td> </tr> </tbody> </table>			Title	Author	Subject	Pages	Issue	Point-Set Topology	Orloff Clark , Lisa	topology	28	No. 41 (Apr. 2016)	Topology Notes	Person, Laura J.	topology	57	No. 40 (Jan. 2016)	Calculus I: Notes and Problems	Stallmann, Cornelius	calculus	76	No. 39 (Sept. 2015)	Discovering Infinity	Olinick, Michael	set theory	71	No.
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Third of three activities:

★ Turn your desks into ~~“V”s~~ ^{chalkboard} and do questions 1–3 of the worksheet I’m about to hand out.

★ After you get started, I’ll designate a “Chalk Czar” .

(\approx 5 minutes)

What you “know” is that the word *derivative* means *slope*, and we write it with a prime: $f'(x)$.



(Time's up.)

Thank you again!!!

Implementing IBL

Developing questions

Assigning groups

Structuring the group work

What you do while they're working

Giving feedback

How to develop questions? Two wide-spread approaches to IBL:

- (Modified) Moore Method

- POGIL

Process-**O**riented, **G**uided **I**nquiry **L**earning

Example of a Moore-Methodesque approach,
in Abstract Algebra

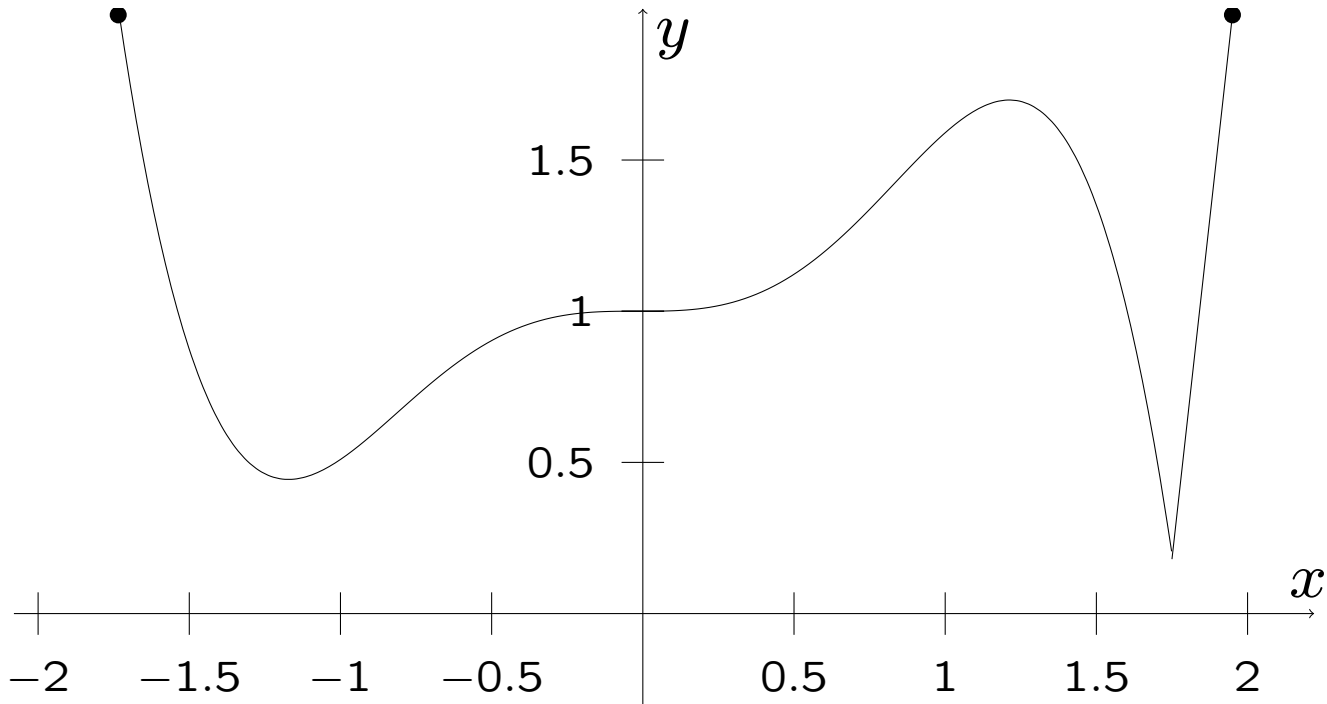
Definition. For any element g in a group G with identity e , we say that $|g|$, the *order* of g , is the smallest positive integer n so that $g^n = e$. If such an integer does not exist, we say $|g| = \infty$.

For each of the statements 1–5 below, either prove the statement or find a counter-example.

1. If G contains an element of order 4, then all elements except the identity have order 4.

2. If G contains an element of order 4, then G contains an element of order 1.
3. If G contains an element of order 4, then G contains an element of order 2.
4. If G contains an element of order 4, then G contains an element of order 3.
5. If G contains an element of order 4, then that is the only element in G with order 4.

Example of a POGIL-esque approach,
in Calculus I



The graph above has three local maxima and two local minima. Circle these five points.

Then fill in the blanks in the following sentence: If f is a function whose domain is the interval $[a, b]$, then local minima and local maxima happen either

- (a) at the _____* of the interval, or
- (b) at places where the derivative is equal to _____,
- or
- (c) at places where the derivative does not _____.

Values of x that satisfy (b) or (c) above are called *critical points* of f .

* *Hint:* Unscramble the letters “d e i n n o p s t”.

How to assign groups?

- with charts
- writing symbols on worksheets
- using questionnaires
- let the students decide

(3–4 people per group seems to work well)

Structure the discussion

- Which problems to work on?
- What roles do students have within groups?
(Chalk Czar!)
- When and how do they stop working?

What the instructor does . . .

- nudge students to answer each other's questions

“What does your group think?”

“Sam wants to know if $x = 2$. Do you all agree?”

“Does everyone in your group have the same answer?”

- assign “ambassadors” from one group to another

- encourage students to take risks

“Put it up on the board! If it's a mistake,

it's a good mistake and everyone will learn from it.”

Feedback and discussion

- Look at the students, not at the board.
- Ask students who didn't put the answer up to explain it.
- Promote risk-taking and learning from “good” mistakes
- Promote “growth” mind-set and effort

Conclusion;

Something witty.

(talking trash? an IBL mind is a terrible thing to waste?)

Thank you.

References

- Brown, Roediger, and McDaniel, *Make it Stick: the science of successful learning*, The Belknap Press of Harvard University Press: London, England, 2014.
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- Richard J. Light, *Making the Most of College*, Harvard University Press: Cambridge MA, 2001.
- Ruder, S.M., & Hunnicutt, S.S. (2008). "POGIL in Chemistry Courses at a Large Urban University: A Case Study," In R.S. Moog, & J.N. Spencer (Eds.), *Process-Oriented Guided Inquiry Learning: ACS Symposium Series 994* (pp. 133–147). Washington, D.C.: American Chemical Society (2008).