Developing Mathematical Creativity: Physics Invention Tasks

Suzanne White Brahmia

Department of Physics

University of Washington
Collaborative Principal Investigators

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Western Washington University

Stephen Kanim;
New Mexico State University

This work is supported by NSF DUE-1045227, NSF DUE-1045231, NSF DUE-1045250
Why do you require physics?
Why do you require physics?

- Dean of School of Pharmacy
- Dean of the School of Engineering
Why do you require physics?

- Dean of School of Pharmacy
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Goal: Students learn to think more like expert physicists
Thinking like a physicist

• Mathematization as a way of reasoning.
• Experimentation as a way of creating knowledge.
Thinking like a physicist

• Mathematization as a way of reasoning.
• Experimentation as a way of creating knowledge.
Mathematization involves...

- representing ideas symbolically,
- defining problems quantitatively,
- producing solutions,
- and checking for coherence.

All in a coordinated effort to understand how the world works.
Do students learn to mathematize through observation?
...they learn recipes:

“There are many occasions when you have to use an equation in Science, particularly in Physics. The Equation Triangles are a way in which you can easily learn to use and rearrange equations, even if you are not confident in your Maths.”
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http://juni.osfc.ac.uk/Extension/level_2_extension/Science/lesson1/equation_triangles.asp
Affective measures reveal counterproductive practices
(from CLASS, 2006, 42 statement survey)

- When I solve a physics problem, I locate an equation that uses the variables given in the problem and plug in the values.

- I do not expect physics equations to help my understanding of the ideas; they are just for doing calculations.

- If I want to apply a method used for solving one physics problem to another problem, the problems must involve very similar situations.
Affective measures: Learning Attitudes Surveys

- CLASS (Adams et al. 2006) U of Colorado, Boulder—typically average of ~10-15% drop in expert-like responses

- MPEX (1998) U of Md –showed systematic deterioration in expertise of student responses regarding the use of math in physics

- The deterioration is less severe in interactive engagement courses.
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Physics is one of the few disciplines in which this kind of mathematical sense-making is essential to its discourse.
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And this mathematization is idiosyncratic and thereby can only be taught by physicists.
Problems

1. Most students leave their introductory college physics course with less expert-like mathematization than before they started.

2. There are disproportionately few African American, Latino and Native American physics majors and graduate students in physics.
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How do successful students mathematize?
Features of successful problem solving

• Bing and Redish (2012) – interplay between formal mathematical manipulation and physical sense-making essential to success
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• Sherin (2001) - flexible and generative understanding of equations is essential
Features of successful problem solving

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• Sherin (2001) - flexible and generative understanding of equations is essential

• Torigoe and Gladding (2012) – reasoning about symbolic representations correlates to course grades
Mathematizing

A **flexible** understanding of equations is essential.

A **generative** use of mathematics is a hallmark of physics for which students have little preparation.

Our discipline has the potential to foster both.
Mathematizing

A **flexible** understanding of equations is essential.

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Our discipline has the potential to foster both.

*But do we?*
How do most students mathematize?
How do most students mathematize?

Obstacles
Concepts in the introductory course are well within a physicists’ limits of mathematization, but are beyond or just at the edge of most students’.

Most instructors have forgotten what it’s like to struggle in this way... have
Rutgers study

• A collection of multiple-choice proportional reasoning items was given as a pretest during the first week of Fall 2013.

• The collection contained 19 items distributed on three pretests in three different subjects (Mechanics, E & M and Chemistry.)
Rutgers study

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- The collection contained 19 items distributed on three pretests in three different subjects (Mechanics, E & M and Chemistry.)
Consider the following statement about Winnie the Pooh’s dream: “There are three times as many heffalumps as woozles.” A correct equation to represent this statement, using $h$ for the number of heffalumps and $w$ for the number of woozles, is:

a. $3h / w$  
 b. $3h = w$  
 c. $3h + w$  
 d. $h = 3w$  
 e. None of these
Heffalumps and woozles

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\( N_{\text{matched}} = 685 \)
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$\sigma_{pooled}=1.8\%$  \hspace{1cm} p-value = 0.8418
Rice Questions

Bartholomew is making rice pudding using his grandmother’s recipe. For three servings of pudding the ingredients include 0.75 pints of milk and 0.5 cups of rice. Bartholomew looks in his refrigerator and sees he has one pint of milk. Given that he wants to use all of the milk, which of the following expressions will help Bartholomew figure out how many cups of rice he should use?

0.5/0.75  
0.75/0.5  
0.5 x 0.75  
(0.5 + 1) x 0.75  
none of these
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Numerical Complexity
*(Calculus-based Intro Mechanics)*

![Graph showing percent correct for different numerical types.]
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p-value < .02
### NJ school math and socioeconomics

*(J. Anyon 1980)*

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| Top 1%       | Executive elite          | Work is developing one’s intellectual powers; **students invent ways to measure and calculate in math class.** |
“The biggest obstacle to success is NOT limitation with math skills or knowing the definition of density...It’s the institutional suppression of thinking.”

-Richard Steinberg 2011
Problems

1. Most students leave their introductory college physics course with less expert-like mathematization than before they started.

2. There are disproportionately few African American, Latino and Native American physics majors and graduate students in physics.
The percentage of the Bachelor’s degrees granted to select underrepresented minorities*

*National Science Foundation’s National Center for Science and Engineering Statistics
The percentage of the Bachelor’s degrees granted to select underrepresented minorities*

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New Jersey 11th Grade Advanced Proficient Math, 2009

Slide courtesy of Michael Marder
On the surface, this seems like a problem with prior math instruction. But it’s not – math in physics has different goals than math in math.
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Physics – flexible and generative mathematics in context
Math – axiomatic reasoning in the absence of context
Somebody else’s problem

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Physics – flexible and generative mathematics in context
Math – axiomatic reasoning in the absence of context

Teaching the mathematical habits of mind that are characteristic of physics thinking should be a major goal of physics instruction at all levels.
• Instructors naturally assume students have a conceptual mastery of arithmetic and algebra.
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• What students master in their math courses is largely procedural, and not conceptual.
Problems

1. Most students leave their introductory college physics course with less expert-like mathematization than before they started.

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Problem

Most physics students, and especially students from low SES high schools, struggle to assimilate the habits of mind we model, and they leave our courses with even less expert-like mathematical attitudes and habits.
We can fix this!
Procedural Mastery
+
Conceptual Understanding
Procedural Mastery

+ 

Conceptual Understanding

Proceptual Understanding
Flexible and generative in early math

(Gray and Tall 1994)

Find 47-35

• **Procedure:** Use number line, start at 47 count left 35 places

• **Process** (Flexibility): Start at 35, move to the right 12 places

• **Proceptual** (Generative): $x=a-b$ represents the mathematical idea “difference”; and $x=a-b$ implies that $a=x+b$
Flexible and generative in early math

\textit{(Gray and Tall 1994)}

Find 47-35

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\textit{Note the foundational thinking for the physics idea of \(\Delta\):}

\(\Delta T=T_f-T_o\) therefore \(T_f=\Delta T + T_o\)
Flexible and generative in early math

*(Gray and Tall 1994)*

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*Note the foundational thinking for the physics idea of $\Delta$:*

$\Delta T = T_f - T_o$ therefore $T_f = \Delta T + T_o$

*comparison  accumulation*
Proceptual divide

The mathematics of flexible procepts is easier than the mathematics of inflexible procedures. The gap is widening because the less successful are actually doing a qualitatively harder form of mathematics.

(Tall 2008)
Proceptual physics
QuanLification as a scientific practice

• relies on a tendency to seek invariance
  ✷ Seeking invariance is at the heart of learning (Gibson & Gibson, 1955).

² Many students don’t spontaneously consider invariance when quantifying nature in school (Simon & Blume, 1994).

² Requires a proceptual understanding of arithmetic

² Tuminaro (2004): Students who do not expect conceptual knowledge of mathematics to connect to physics problems do not engage in sense making when calculating.

² Brahmia & Boudreaux (2016): Students’ errors can be traced to a failure to distinguish products from factors when reasoning about physics quantiles.
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  ✦ Brahmia & Boudreaux (2016): Students errors can be traced to a failure to distinguish products from factors when reasoning about physics quantities.
Your task this time is to come up with a **fastness index** for cars with dripping oil. All the cars drip oil once a second.

**This task is a little harder than before.** A company always makes its cars go the same fastness. We will not tell you how many companies there are. You have to decide which cars are from the same company. They may look different!
Quantification is a conceptual blend

*double scope arithmetic reasoning blend*, in which two distinct domains of thinking are merged to form a new cognitive space optimally suited for productive work.
ICC (Inventing with Contrasting Cases)

Schwartz, Chase, Oppezzo, & Chin 2011

• Instructional model designed to help students develop the tendency to
  – Seek invariance
  – Make sense with compound quantities
  – Contrasting helps students notice what matters and what doesn’t
  – Preparation for subsequent instruction
Invention Instruction

**Starting Resources**
- Math procedures (disconnected)
- Capacity to respond to prompts to calculate (rigid response)
- Disconnected definitions of some physics concepts

**Invention Tasks**
*(quantification and symbolizing)*

**Coordinated set of Resources**
- Proceptual understanding of mathematics
- Flexibility in mathematizing
- Capacity to invent or imagine inventing physical quantities
Applying ICC: Physics Invention Tasks

**QF:** Identify quantifiable features of the system

**AC:** Arithmetically construct index

**CC:** Compare across contrasting cases for invariance

**MU:** Make meaning of units

**RC:** Rules and constraints imposed by the invention

**CM:** Clarify Mission-
To make mathematical choices to generate a useful quantity

_Harel's necessity principle_

**NC:** Evaluate in new context

**Collaborative productive failure**
_Socioconstructivist framework_
These cars all drip oil once every second. Invent a **speeding-up index** that allows you to rank the cars in terms of how quickly they speed up.

Sample Invention Sequence 1

<table>
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<th>Car B</th>
<th>Car C</th>
<th>Car D</th>
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<tbody>
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<td>5mph</td>
<td>0</td>
<td>10mph</td>
<td>4mph</td>
<td>12mph</td>
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<td>9mph</td>
<td>55mph</td>
<td>34mph</td>
<td>57mph</td>
</tr>
</tbody>
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Sociocultural Benefits

• Valuing naïve understanding *(Ross & Otero 2013)*

• Shifting authority from instructor to social consensus *(Ross & Otero 2013)*
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• Valuing naïve understanding (Ross & Otero 2013)

• Shifting authority from instructor to social consensus (Ross & Otero 2013)

• Addressing stereotype threat: Not remediation; students work, and struggle, collaboratively. (Steele & Aronson 1995)
Sociocultural Benefits

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• Shifting authority from instructor to social consensus (Ross & Otero 2013)

• Addressing stereotype threat: Not remediation; students work, and struggle, collaboratively. (Steele & Aronson 1995)

• Developing self-efficacy: Invention process gives ownership of the knowledge to the student (Bandura 1997, Sawtelle 2011)
FCI comparison
(before the introduction of PITs, 2003, n=102 and after 2013/14, n=144)
CLASS- physics categories associated with mathematical reasoning, pre-instruction and the gains over one semester.

Combined Fall 2013 and Fall 2014, n=121.

Error bars represent the standard error.
Rutgers Engineering Physics Study

• Underprepared (precalc math placement) vs Mainstream (calculus math placement)
Rutgers Engineering Physics Study

• Underprepared (precalc math placement) vs Mainstream (calculus math placement)

• Simultaneous courses
Rutgers Engineering Physics Study

- Underprepared (precalc math placement) vs Mainstream (calculus math placement)
- Simultaneous courses
- Same content, different curricula
Rutgers Engineering Physics Study

• Underprepared (precalc math placement) vs Mainstream (calculus math placement)

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• Same content, different curricula

• FCI, Math reasoning, and CLASS pre/post Fall 2013
# Course Demographic Comparison

<table>
<thead>
<tr>
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<th>EAP I (Underprepared)</th>
<th>AP I (Mainstream)</th>
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</thead>
<tbody>
<tr>
<td># of students</td>
<td>~120</td>
<td>~700</td>
</tr>
<tr>
<td>Mean SAT</td>
<td>610</td>
<td>680</td>
</tr>
<tr>
<td>% URM</td>
<td>40%</td>
<td>12%</td>
</tr>
<tr>
<td>% female</td>
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<td>21%</td>
</tr>
<tr>
<td>Median MHI of sending district</td>
<td>0.7*Q (p\text{-value}&lt;.000000001)</td>
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Force Concept Inventory; $\sigma_{\text{mean}}$: EAP I (n=135) 1.4%(pre), 1.5%(post); AP I (n=757) 0.8%(pre), 0.8%(post)
CLASS

While the EAP course shows small positive gains, the AP course shows negative gains ~10% across PS categories.
A bicycle is equipped with an odometer to measure how far it travels. A cyclist rides the bicycle up a mountain road. When the odometer reading increases by 8 miles, the cyclist gains $H$ vertical feet of elevation. Find an expression for the number of miles the odometer reading increases for every vertical foot of elevation gain.

\[
\sin^{-1}\left( \frac{8}{H} \right) \quad \sin^{-1}\left( \frac{H}{8} \right) \quad \frac{H}{8} \quad \frac{8}{H} \quad \text{None of these}
\]
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\[
\sin^{-1}\left(\frac{8}{H}\right) \quad \sin^{-1}\left(\frac{H}{8}\right) \quad H/8 \quad 8/H \quad \text{None of these}
\]
Bike Path RU Fall 2013
One semester of instruction

pre
post
Bike Path RU (full year of instruction) \[ n_{115/6} = 187 \text{ and } n_{123/4} = 583 \]
Rice Questions (SES)
(full year)

- APIhigh
- APIlow
- EAPIlow
CLASS Problem Solving - General

% that agree with experts

SES: APIHi, APILow
% that agree with experts

CLASS Problem Solving - General

SES:  ➡️ APIHi  ➩ APILow  ➢ EAPILow
CLASS Personal Interest

Percentage agree with experts

Pre

Post

SES: APIHi, APILow
Thank you!
Physics Invention Tasks website:

http://faculty.uw.edu/pits

Password (case sensitive): Treehouse